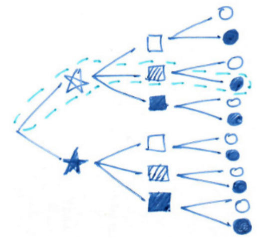
Counting

Multiplication rule and tree diagrams



Symbol	Number of types of
$n_{☆}$	stars
$n_{□}$	boxes for each star
$n_{○}$	disks for each box

Number of leaves

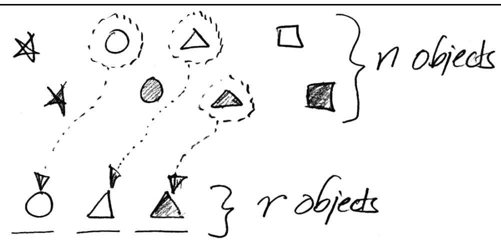
$$= n_{☆} \cdot n_{□} \cdot n_{○}$$

Factorial

$$n! := n \cdot (n-1) \cdot (n-2) \cdot \cdots \cdot 2 \cdot 1$$

$$0! := 1$$

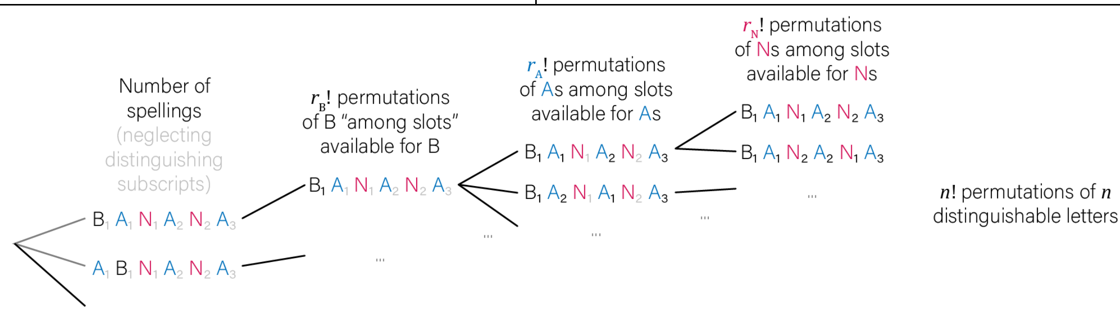
Permutations



Number of distinct ordered arrangements that can be produced by placing r of n distinct objects in r distinct locations.

$$_nP_r = \frac{n!}{(n-r)!}$$

Permutations with repetitions



Number of distinct ordered arrangements of n objects, r_1 of which are indistinguishable copies of one item, r_2 of which are indistinguishable copies of another item, etc.

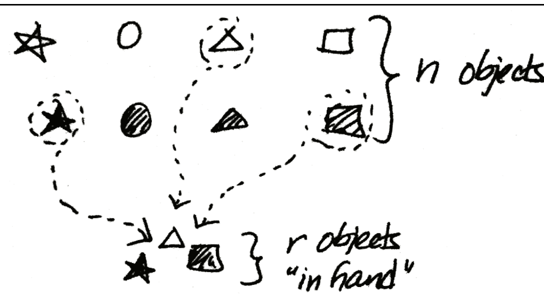
$$\frac{n!}{r_1!\, r_2! \cdots}$$

Combinations (interpretation I: arrangements of stars and balls)

Number of ways to arrange k stars and $n-k$ balls in n distinct locations

$$_nC_k = \frac{n!}{(n-k)!\, k!}$$

Combinations (interpretation II: objects in hand)



Number of ways to grab r objects in hand from among n distinct objects.

$$_nC_r = \frac{n!}{(n-r)!\, r!}$$

Using combinations (interpretation III: stars in boxes)

k stars in n distinguishable boxes:

[☆☆] [☆☆☆] [☆]

k stars and $n+1$ walls:

|☆☆|☆☆☆|☆|

Number of ways to allocate k indistinguishable stars among n distinguishable boxes

$$_{k+(n+1)-2}C_k$$